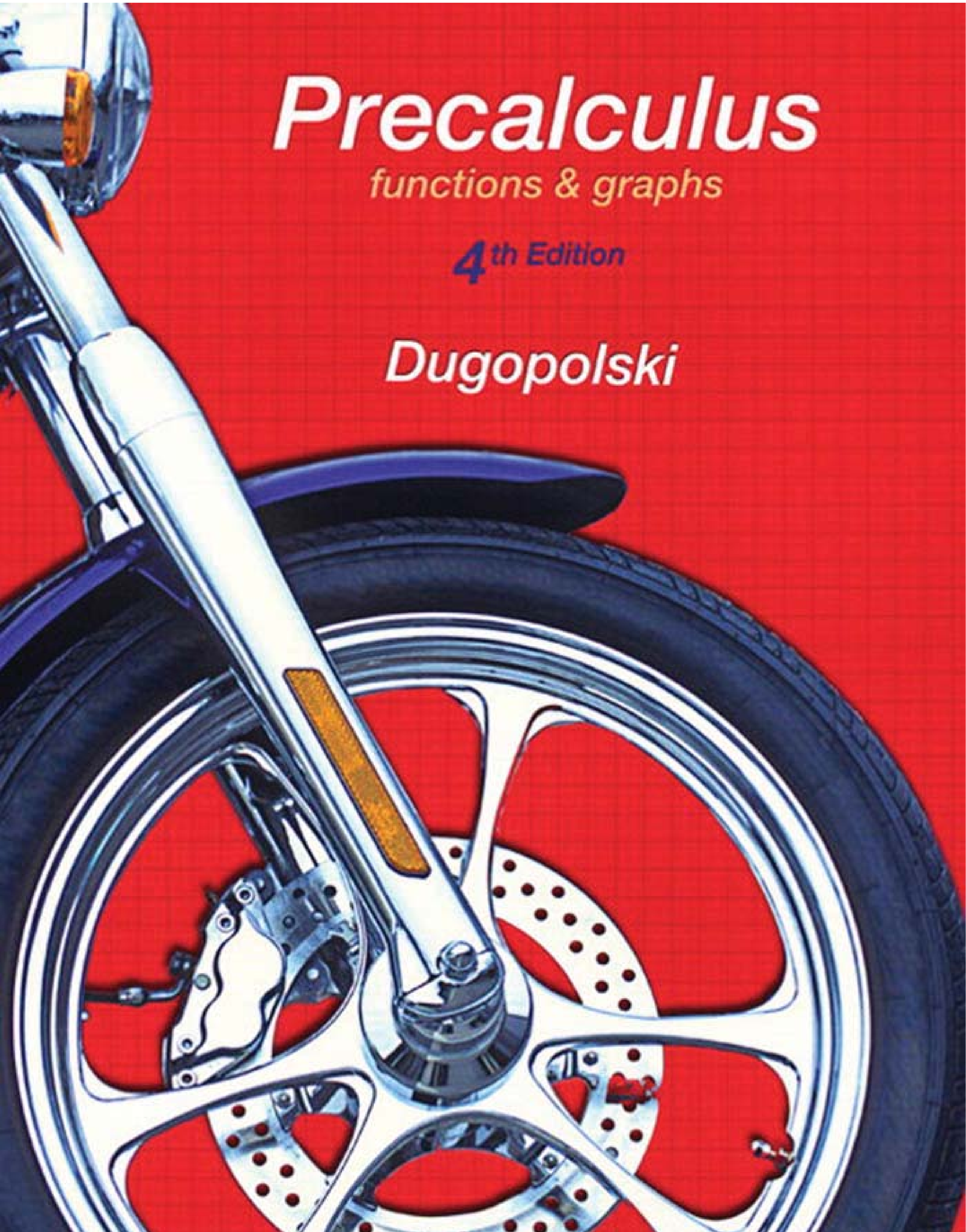
Precalculus

functions & graphs

4th Edition

Dugopolski



Precalculus: Functions and Graphs

4th Edition

Mark Dugopolski

Southeastern Louisiana University

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Preface

Making the transition to calculus is more than just finding a new classroom: it's about being prepared to grasp bigger and more complex mathematical concepts. *Precalculus: Functions and Graphs* is designed to make this transition seamless by focusing on all the skills that will be needed to succeed in calculus and beyond. This text contains the rigor essential for building a strong foundation of mathematical skills and concepts, and at the same time supports students' mathematical needs with a number of tools newly developed for this revision. With an eye toward future courses, this text provides students with an excellent opportunity to sharpen their critical thinking skills and introduces them to the usefulness and applicability of mathematics beyond the classroom. It is my goal that students will benefit from this approach and find that the mathematics presented here will enrich their classroom experience in this course as well as in future mathematics courses. Preparation is the foundation for success, and *Precalculus: Functions and Graphs* will help you succeed in this course and beyond.

New to the Fourth Edition

For this edition of *Precalculus: Functions and Graphs*, I have updated explanations, examples, exercises, and art in response to comments from users of the last edition. In particular, I have written more than 900 new exercises that are specifically designed to increase student understanding and retention of the concepts that are taught in this text. Here are the major changes in this edition.

- **Updated real-data** in examples, exercises, and chapter openers make the text relevant for today's students.
- **Fill-in-the-blank exercises** are now used at the beginning of the section exercise sets to help students learn definitions, rules, and theorems.
- **Cumulative review exercises** are now used at the end of the section exercise sets to keep current the skills learned in previous sections and chapters. These exercises are under the heading "Rethinking."
- **Tying It All Together** exercises have been expanded to include fill-in-the-blank exercises that emphasize vocabulary.
- **Polynomial and rational inequalities** are now solved with one method, the test-point method. This change simplifies the procedure and makes it more consistent with techniques used in calculus.
- **Limit notation** is now introduced and used to describe the asymptotic behavior of exponential, logarithmic, rational, and trigonometric functions.
- **Try This** exercises have been included after every example in the text. These exercises are very similar to the corresponding examples and give the students the opportunity to immediately check their understanding of the example. Solutions to all of the Try This exercises are in the appendix of the *Student Edition*.
- **Suggested homework problems** are indicated in the Annotated Instructor's Edition. Each section exercise set contains 20 to 24 underlined exercise numbers. These exercises can be used as a set of homework exercises for the section.

Continuing Features

With each new edition, all of the features are reviewed to make sure they are providing a positive impact on student success. The continuing features of the text are listed here.

Strategies for Success

- **Chapter Opener** Each chapter begins with chapter opener text that discusses a real-world situation in which the mathematics of the chapter is used. Examples and exercises that relate back to the chapter opener are included in the chapter.
- **Foreshadowing Calculus** This feature gives a brief indication of the connection between certain algebraic topics and calculus, and identifies topics that will be continued in calculus.
- **Summaries** of important concepts are included to help students clarify ideas that have multiple parts.
- **Strategies** contain general guidelines for solving certain types of problems. They are designed to help students sharpen their problem-solving skills.
- **Procedures** are similar to *Strategies*, but are more specific and more algorithmic. *Procedures* are designed to give students a step-by-step approach for solving a specific type of problem.
- **Function Galleries** Located throughout the text, these function summaries are also gathered together at the beginning of the text. These graphical summaries are designed to help students link the visual aspects of various families of functions to the properties of the functions.
- **Historical Notes** Located in the margins throughout the text, these brief essays are designed to connect the topics of precalculus to the mathematicians who first studied them and to give precalculus a human face.
- **Hints** Selected applications include hints that are designed to encourage students to start thinking about the problem at hand. A *Hint* logo HINT is used where a hint is given.
- **Graphing Calculator Discussions** Optional graphing calculator discussions have been included in the text. They are clearly marked by graphing calculator icons so that they can be easily skipped if desired. Although the graphing calculator discussions are optional, all students will benefit from reading them. In this text, the graphing calculator is used as a tool to support and enhance algebraic conclusions, not to make conclusions.

Section Exercises and Review

- **For Thought** Each exercise set is preceded by a set of ten true/false questions that review the basic concepts in the section, help check student understanding, and offer opportunities for writing and discussion. The answers to all *For Thought* exercises are included in the back of the *Student Edition*.
- **Exercise Sets** The exercise sets range from easy to challenging, and are arranged in order of increasing difficulty. Those exercises that require a graphing calculator are optional and are marked with an icon.
- **Writing/Discussion and Cooperative Learning Exercises** These exercises deepen students' understanding by giving them the opportunity to express mathematical ideas both in writing and to their classmates during small group or team discussions.
- **Thinking Outside the Box** Found throughout the text, these problems are designed to get students and instructors to do some mathematics just for fun. I enjoyed solving these problems and hope that you will too. The problems can be used for individual or group work. They may or may not have anything to do with the sections in which they are located and might not even require any techniques discussed in the text. So be creative and try *Thinking Outside the Box.* The answers are given in the *Annotated Instructor's Edition* only, and complete solutions can be found in the *Instructor's Solutions Manual.*
- **Pop Quizzes** Included at the end of every section of the text, the *Pop Quizzes* give instructors and students convenient quizzes of 8 to 10 questions that can be used in the classroom to check understanding of the basics. The answers appear in the *Annotated Instructor's Edition* only.

- **Linking Concepts** This feature is located at the end of nearly every exercise set. It is a multipart exercise or exploration that can be used for individual or group work. The idea of this feature is to use concepts from the current section along with concepts from preceding sections or chapters to solve problems that illustrate the links among various ideas. Some parts of these questions are open-ended, and require somewhat more thought than standard skill-building exercises. Answers are given in the *Annotated Instructor's Edition* only, and full solutions can be found in the *Instructor's Solutions Manual.*

Chapter Review

- **Highlights** This end-of-chapter feature contains an overview of all of the concepts presented in the chapter, along with brief examples to illustrate the concepts.
- **Chapter Review Exercises** These exercises are designed to give students a comprehensive review of the chapter without reference to individual sections and to prepare students for a chapter test.
- **Chapter Test** The problems in the *Chapter Test* are designed to measure the student's readiness for a typical one-hour classroom test. Instructors may also use them as a model for their own end-of-chapter tests. Students should be aware that their in-class test could vary from the *Chapter Test* due to different emphasis placed on the topics by individual instructors.
- **Tying It All Together** Found at the end of most chapters in the text, these exercises help students review selected concepts from the present and prior chapters, and require students to integrate multiple concepts and skills.
- **Concepts of Calculus** Most chapters end with a discussion of a particular concept of calculus, as well as exercises designed to illustrate that concept. This one-page feature will give students a preview of important topics of calculus, and may be used as a writing or collaborative learning assignment.
- **Index of Applications** The many applications contained within the text are listed in the *Index of Applications* that appears at the end of the text. The applications are page referenced and grouped by subject matter.

Acknowledgments

Thanks to all of the professors and students who have used this text in previous editions. Special thanks to those who have taken the time to send me their comments and suggestions. I am always glad to hear from users of my texts. You can e-mail me at bookinit@charter.net. Thanks also to the professors who have reviewed all of the previous editions of this text.

Thanks to Edgar N. Reyes, Southeastern Louisiana University, for working all of the exercises and writing the *Solutions Manuals* and to Rebecca W. Muller, Southeastern Louisiana University, for writing the *Instructor's Testing Manual*. I wish to express my thanks to John Morin and Tom Wegleitner for accuracy checking. A special thanks to Nesbitt Graphics, the compositor, for the superb work they did on this book.

Finally, it has been another wonderful experience with the talented and dedicated Pearson team.

As always, thanks to my wife, Cheryl, whose love, encouragement, understanding, support, and patience are invaluable.

Mark Dugopolski
Ponchatoula, Louisiana

Supplements List

Student Supplements

Student's Solutions Manual

- By Edgar N. Reyes, *Southeastern Louisiana University*.
- Provides detailed solutions to all odd-numbered text exercises.
- ISBN-13: 978-0-321-79118-4; ISBN-10: 0-321-79118-5

A Review of Algebra

- By Heidi Howard, *Florida Community College at Jacksonville*.
- Provides additional support for those students needing further algebra review.
- ISBN-13: 978-0-201-77347-7; ISBN-10: 0-201-77347-3

Instructor Supplements

Annotated Instructor's Edition

- All answers included, with answers to most exercises on the page where they occur.
- Groups of exercises are keyed back to corresponding examples from the section.
- ISBN-13: 978-0-321-79063-7; ISBN-10: 0-321-79063-4

Instructor's Solutions Manual

- By Edgar N. Reyes, *Southeastern Louisiana University*.
- Provides complete solutions to all text exercises, including the *For Thought* and *Linking Concepts* exercises.
- ISBN-13: 978-0-321-79117-7; ISBN-10: 0-321-79117-7

Instructor's Testing Manual

- By Rebecca W. Muller, *Southeastern Louisiana University.*
- Contains six alternative forms of tests per chapter (two are multiple choice).
- Answer keys are included.
- Available for download from Pearson Education's online catalog.

Graphing Calculator Manual

- By Darryl Nester, *Bluffton University.*
- Provides instructions and keystroke operations for the TI-83 Plus, TI-84 Plus, and TI-89.
- Also contains worked-out examples taken directly from the text.
- Available for download from Pearson Education's online catalog.

PowerPoint Lecture Presentation

- Classroom presentation slides are geared specifically to sequence this textbook.
- Available within MyMathLab or from Pearson Education's online catalog.

Technology Resources

MathXL® Online Course (access code required)

MathXL® is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning management system.) With MathXL, instructors can:

- Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- Create and assign their own online exercises and import TestGen tests for added flexibility.
- Maintain records of all student work tracked in MathXL's online gradebook.

With MathXL, students can:

- Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.
- Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.
- Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com, or contact your Pearson representative.

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from a **trusted partner** with educational expertise and an eye on the future.

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To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

MathXL® Tutorials on CD

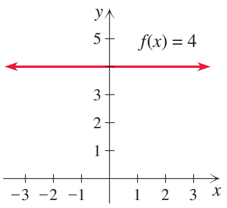
This interactive tutorial CD-ROM provides algorithmically generated practice exercises that are correlated at the objective level to the exercises in the textbook. Every practice exercise is accompanied by an example and a guided solution designed to involve students in the solution process. Selected exercises may also include a video clip to help students visualize concepts. The software provides helpful feedback for incorrect answers and can generate printed summaries of students' progress.

TestGen®

TestGen® (www.pearsonhighered.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. Tests can be printed or administered online. The software and testbank are available for download from Pearson Education's online catalog.

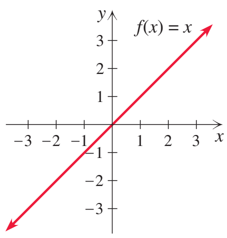
FUNCTION gallery... Some Basic Functions and Their Properties

Constant Function



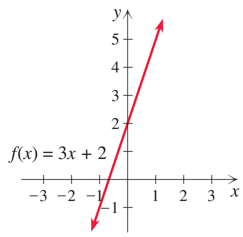
Domain $(-\infty, \infty)$
Range $\{4\}$
Constant on $(-\infty, \infty)$
Symmetric about *y*-axis

Identity Function



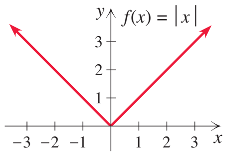
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

Linear Function



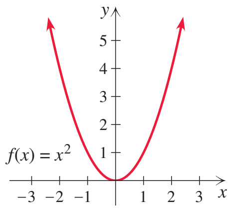
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$

Absolute Value Function



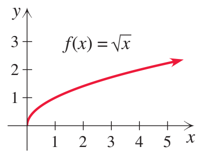
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Increasing on $[0, \infty)$
Decreasing on $(-\infty, 0]$
Symmetric about *y*-axis

Square Function



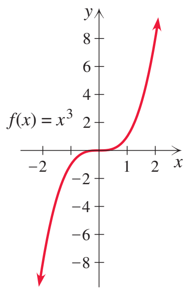
Domain $(-\infty, \infty)$
Range $[-0, \infty)$
Increasing on $[0, \infty)$
Decreasing on $(-\infty, 0]$
Symmetric about *y*-axis

Square-Root Function



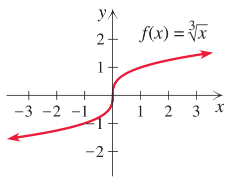
Domain $[0, \infty)$
Range $[0, \infty)$
Increasing on $[0, \infty)$

Cube Function



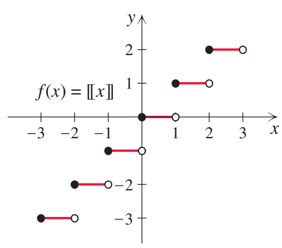
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

Cube-Root Function



Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

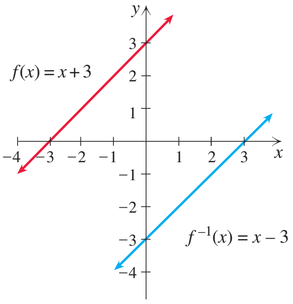
Greatest Integer Function

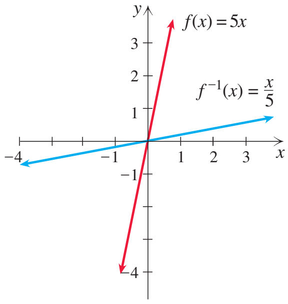


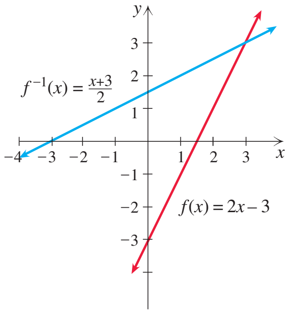
Domain $(-\infty, \infty)$
Range $\{n \mid n \text{ is an integer}\}$
Constant on $[n, n + 1)$
for every integer n

FUNCTION gallery... Some Inverse Functions

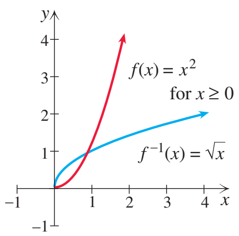
Linear

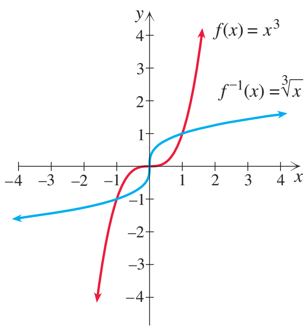


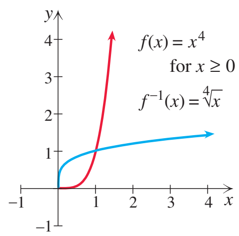




Powers and Roots

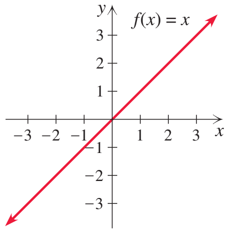




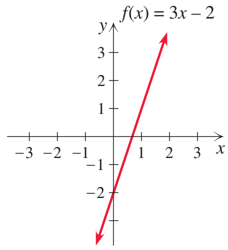


FUNCTION gallery... Polynomial Functions

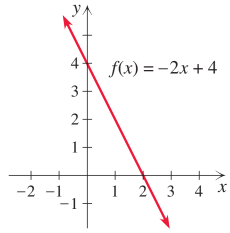
Linear: $f(x) = mx + b$



Slope 1, y-intercept $(0, 0)$

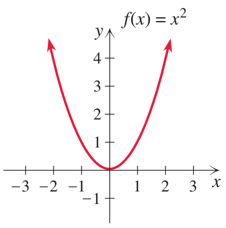


Slope 3, y-intercept $(0, -2)$

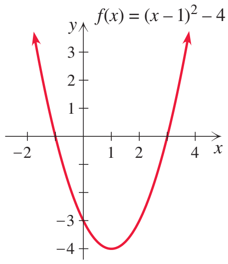


Slope -2, y-intercept $(0, 4)$

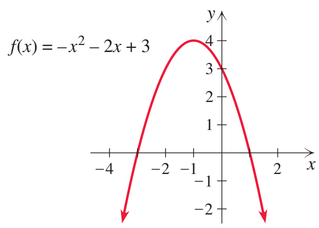
Quadratic: $f(x) = ax^2 + bx + c$ or $f(x) = a(x - h)^2 + k$



Vertex $(0, 0)$
Range $[0, \infty)$

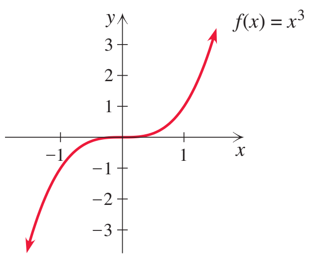


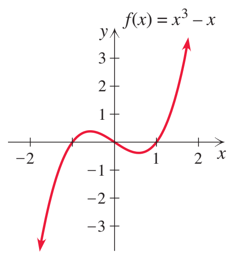
Vertex $(1, -4)$
Range $[-4, \infty)$

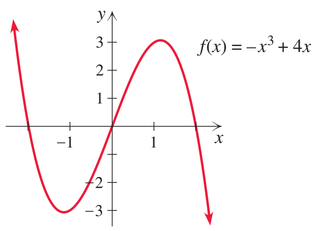


Vertex $(-1, 4)$
Range $(-\infty, 4]$

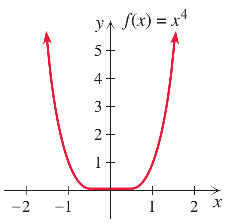
Cubic: $f(x) = ax^3 + bx^2 + cx + d$

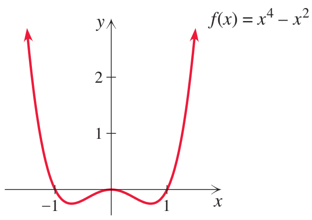


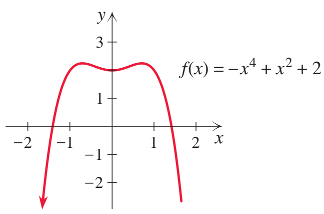




Quartic or Fourth-Degree: $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

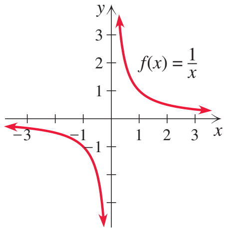


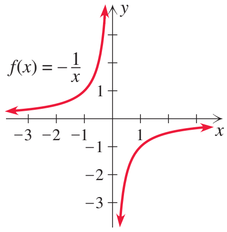


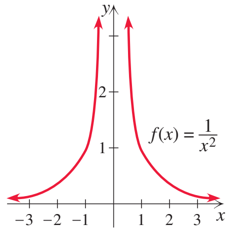


FUNCTION gallery... Some Basic Rational Functions

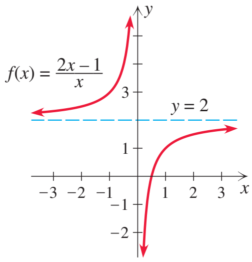
Horizontal Asymptote *x*-axis and Vertical Asymptote *y*-axis

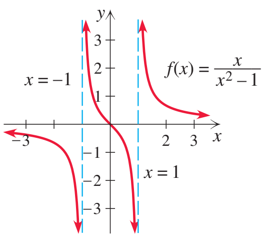


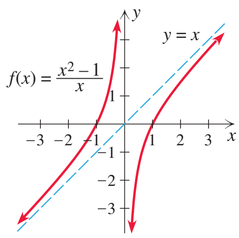




Various Asymptotes

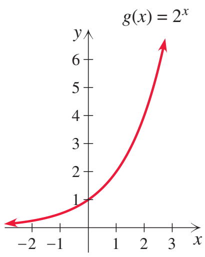




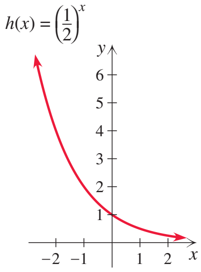


FUNCTION gallery... Exponential and Logarithmic Functions

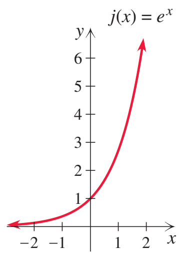
Exponential: $f(x) = a^x$, domain $(-\infty, \infty)$, range $(0, \infty)$



Increasing on $(-\infty, \infty)$
y-intercept $(0, 1)$

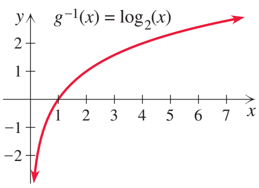


Decreasing on $(-\infty, \infty)$
y-intercept $(0, 1)$

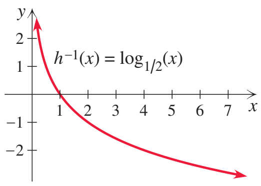


Increasing on $(-\infty, \infty)$
y-intercept $(0, 1)$

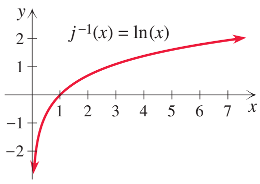
Logarithmic: $f^{-1}(x) = \log_a(x)$, domain $(0, \infty)$, range $(-\infty, \infty)$



Increasing on $(0, \infty)$
x-intercept $(1, 0)$

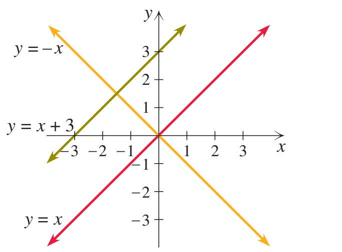


Decreasing on $(0, \infty)$
x-intercept $(1, 0)$

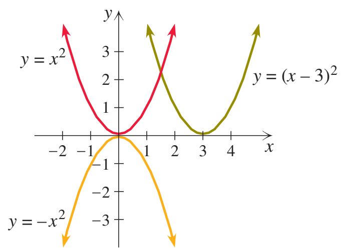


Increasing on $(0, \infty)$
x-intercept $(1, 0)$

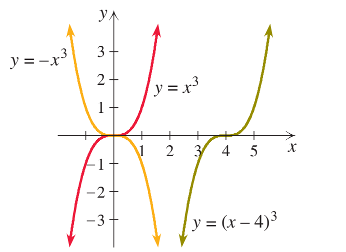
FUNCTION gallery... Some Basic Functions of Algebra with Transformations



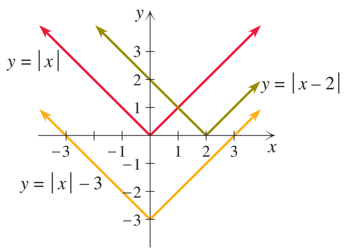
Linear



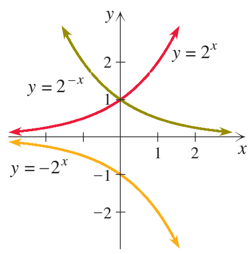
Quadratic



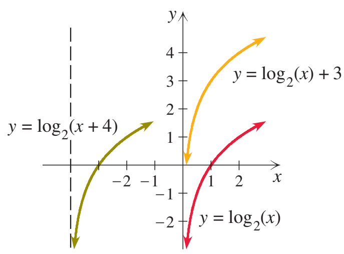
Cubic



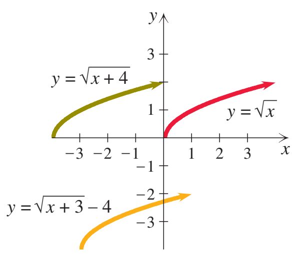
Absolute value



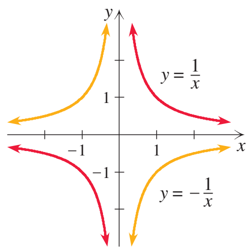
Exponential



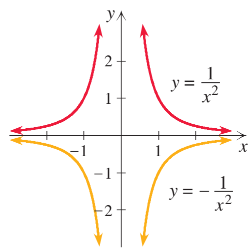
Logarithmic



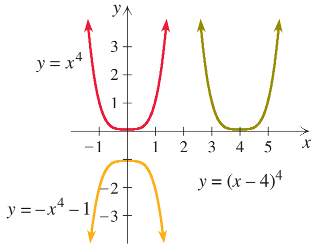
Square root



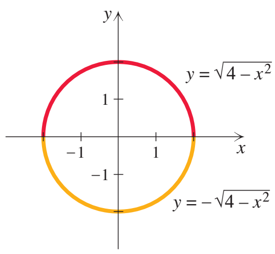
Reciprocal



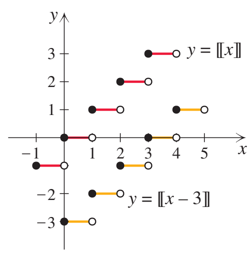
Rational



Fourth degree

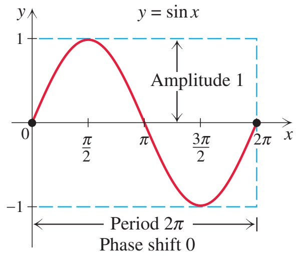


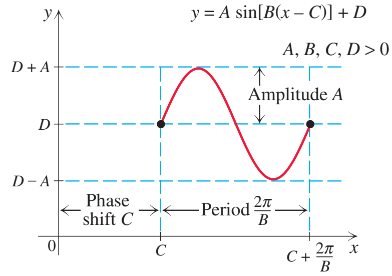
Semicircle

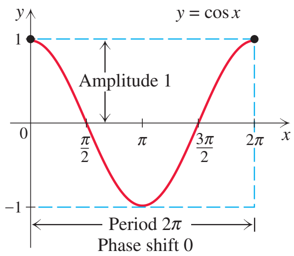


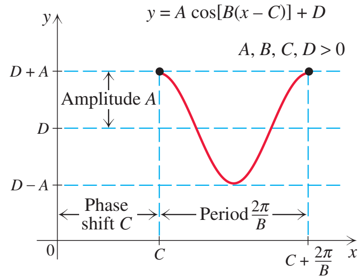
Greatest integer

FUNCTION gallery... The Sine and Cosine Functions

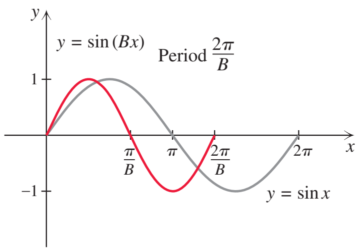


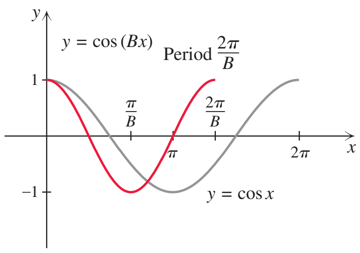


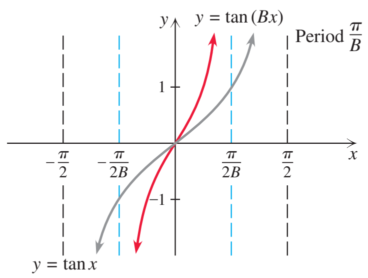




FUNCTION gallery... Periods of Sine, Cosine, and Tangent $(B > 1)$

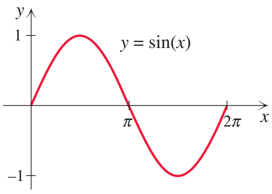


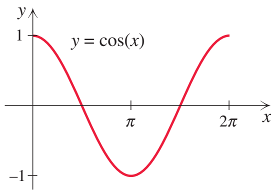


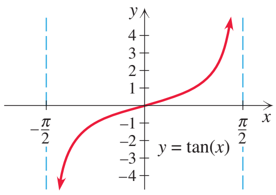


Fundamental cycles

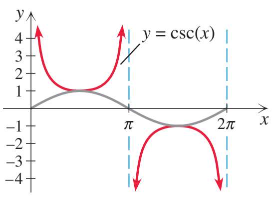
FUNCTION gallery... Trigonometric Functions

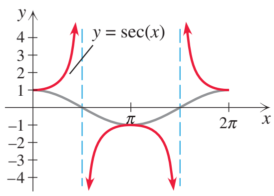


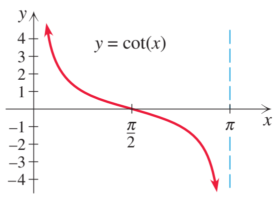




Domain (k any integer)	$(-\infty, \infty)$	$(-\infty, \infty)$	$x \neq \frac{\pi}{2} + k\pi$
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Period	2π	2π	π
Fundamental cycle	$[0, 2\pi]$	$[0, 2\pi]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

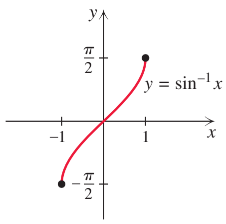




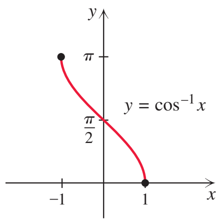


Domain (k any integer)	$x \neq k\pi$	$x \neq \frac{\pi}{2} + k\pi$	$x \neq k\pi$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, \infty)$
Period	2π	2π	π
Fundamental cycle	$[0, 2\pi]$	$[0, 2\pi]$	$[0, \pi]$

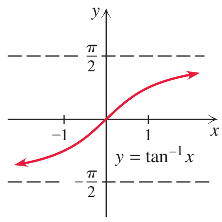
FUNCTION gallery... Inverse Trigonometric Functions



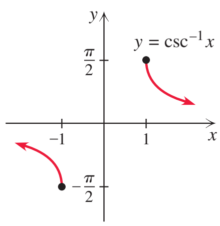
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



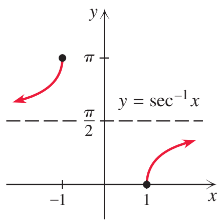
Domain $[-1, 1]$
Range $[0, \pi]$



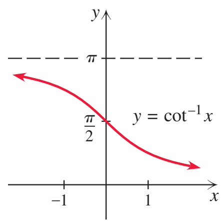
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

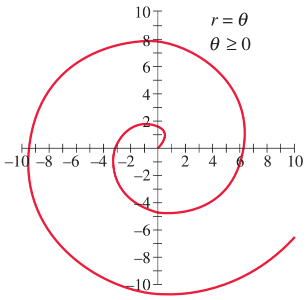


Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

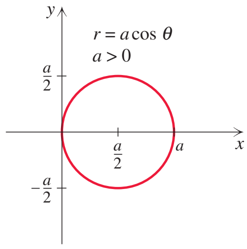


Domain $(-\infty, \infty)$
Range $(0, \pi)$

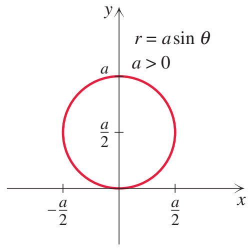
FUNCTION gallery... Functions in Polar Coordinates



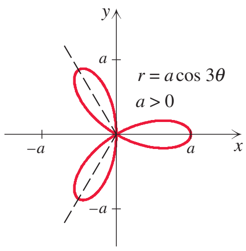
Spiral



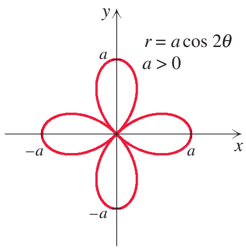
Circle



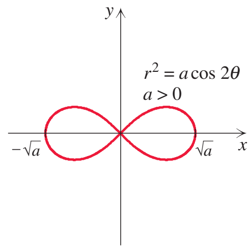
Circle



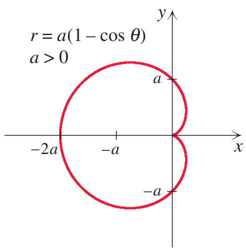
Three-Leaf Rose



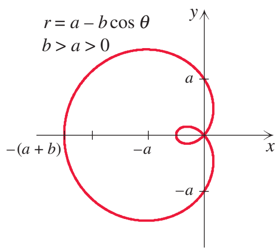
Four-Leaf Rose



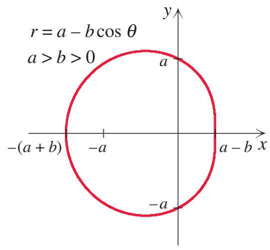
Lemniscate



Cardioid



Limaçon



Limaçon

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1 Equations, Inequalities, and Modeling

Even infamous Heartbreak Hill couldn't break the winning spirit of Ethiopian runner Dire Tune as she focused on first place. It was the 112th running of the Boston Marathon, a 26.2-mile ordeal that one runner called "14 miles of fun, 8 miles of sweat, and 4 miles of hell!"

Sporting events like the 2008 Marathon have come a long way since the first Olympic Games were held over 2500 years ago. Today, sports and science go hand in hand. Modern athletes often use mathematics to analyze the variables that help them increase aerobic capacity, reduce air resistance, or strengthen muscles.

WHAT YOU WILL learn... In this chapter you will see examples of sports applications while you learn to solve equations and inequalities. By the time you reach the finish line, you will be using algebra to model and solve problems.

1.1 Equations in One Variable

1.2 Constructing Models to Solve Problems

1.3 Equations and Graphs in Two Variables

1.4 Linear Equations in Two Variables

1.5 Scatter Diagrams and Curve Fitting

1.6 Complex Numbers

1.7 Quadratic Equations

1.8 Linear and Absolute Value Inequalities

1.1 Equations in One Variable

One of our main goals in algebra is to develop techniques for solving a wide variety of equations. In this section we will solve linear equations and other similar equations.

Definitions

An **equation** is a statement (or sentence) indicating that two algebraic expressions are equal. The verb in an equation is the equality symbol. For example, $2x + 8 = 0$ is an equation. If we replace x by -4, we get $2 \cdot (-4) + 8 = 0$, a true statement. So we say that -4 is a **solution** or **root** to the equation or -4 **satisfies** the equation. If we replace x by 3, we get $2 \cdot 3 + 8 = 0$, a false statement. So 3 is not a solution.

Whether the equation $2x + 8 = 0$ is true or false depends on the value of x, and so it is called an **open sentence.** The equation is neither true nor false until we choose a value for x. The set of all solutions to an equation is called the **solution set** to the equation. To **solve** an equation means to find the solution set. The solution set for $2x + 8 = 0$ is $\{-4\}$. The equation $2x + 8 = 0$ is an example of a linear equation.

Definition: Linear Equation in One Variable

A **linear equation in one variable** is an equation of the form $ax + b = 0$, where a and b are real numbers, with $a \neq 0$.

Note that other letters can be used in place of x. For example, $3t + 5 = 0$, $2w - 6 = 0$, and $-2u + 7 = 0$ are linear equations.

Solving Linear Equations

The equations $2x + 8 = 0$ and $2x = -8$ both have the solution set $\{-4\}$. Two equations with the same solution set are called **equivalent** equations. Adding the same real number to or subtracting the same real number from each side of an equation results in an equivalent equation. Multiplying or dividing each side of an equation by the same nonzero real number also results in an equivalent equation. These **properties of equality** are stated in symbols in the following box.

Properties of Equality

If A and B are algebraic expressions and C is a real number, then the following equations are equivalent to $A = B$:

$A + C = B + C$	**Addition property of equality**
$A - C = B - C$	**Subtraction property of equality**
$CA = CB \; (C \neq 0)$	**Multiplication property of equality**
$\frac{A}{C} = \frac{B}{C} \; (C \neq 0)$	**Division property of equality**

We can use an algebraic expression for C in the properties of equality, because the value of an algebraic expression is a real number. However, this can produce nonequivalent equations. For example,

$$x = 0 \qquad \text{and} \qquad x + \frac{1}{x} = 0 + \frac{1}{x}$$

appear to be equivalent by the addition property of equality. But the first is satisfied by 0 and the second is not. When an equation contains expressions that are undefined for some real number(s) then we must check all solutions carefully.

Any linear equation, $ax + b = 0$, can be solved in two steps. Subtract b from each side and then divide each side by a $(a \neq 0)$, to get $x = -b/a$. Although the equations in our first example are not exactly in the form $ax + b = 0$, they are often called linear equations because they are equivalent to linear equations.

EXAMPLE 1 Using the properties of equality

Solve each equation.

a. $3x - 4 = 8$ **b.** $\frac{1}{2}x - 6 = \frac{3}{4}x - 9$ **c.** $3(4x - 1) = 4 - 6(x - 3)$

Solution

a.

$$3x - 4 = 8$$

$$3x - 4 + 4 = 8 + 4 \quad \text{Add 4 to each side.}$$

$$3x = 12 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{12}{3} \quad \text{Divide each side by 3.}$$

$$x = 4 \quad \text{Simplify.}$$

Since the last equation is equivalent to the original, the solution set to the original equation is $\{4\}$. We can check by replacing x by 4 in $3x - 4 = 8$. Since $3 \cdot 4 - 4 = 8$ is correct, we are confident that the solution set is $\{4\}$.

b. Multiplying each side of the equation by the least common denominator, LCD, will eliminate all of the fractions:

$$\frac{1}{2}x - 6 = \frac{3}{4}x - 9$$

$$4\left(\frac{1}{2}x - 6\right) = 4\left(\frac{3}{4}x - 9\right) \quad \text{Multiply each side by 4, the LCD.}$$

$$2x - 24 = 3x - 36 \quad \text{Distributive property}$$

$$2x - 24 - 3x = 3x - 36 - 3x \quad \text{Subtract } 3x \text{ from each side.}$$

$$-x - 24 = -36 \quad \text{Simplify.}$$

$$-x = -12 \quad \text{Add 24 to each side.}$$

$$(-1)(-x) = (-1)(-12) \quad \text{Multiply each side by } -1.$$

$$x = 12 \quad \text{Simplify.}$$

Check 12 in the original equation. The solution set is $\{12\}$.

c. $3(4x - 1) = 4 - 6(x - 3)$

$$12x - 3 = 4 - 6x + 18 \quad \text{Distributive property}$$

$$12x - 3 = 22 - 6x \quad \text{Simplify.}$$

$$18x - 3 = 22 \quad \text{Add } 6x \text{ to each side.}$$

$$18x = 25 \quad \text{Add 3 to each side.}$$

$$x = \frac{25}{18} \quad \text{Divide each side by 18.}$$

Check 25/18 in the original equation. The solution set is $\left\{\frac{25}{18}\right\}$.

You can use a graphing calculator to calculate the value of each side of the equation when x is 25/18 as shown in Fig. 1.1.

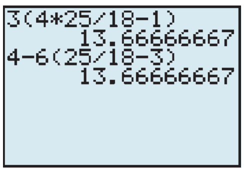

Figure 1.1

▶**TRY THIS.** Solve $5(3x - 2) = 5 - 7(x - 1)$. ■

Note that checking the equation in Example 1(c) with a calculator did not prove that 25/18 is the correct solution. The properties of equality that were applied correctly in each step guarantee that we have the correct solution. The values of the two sides of the equation could agree for the 10 digits shown on the calculator and disagree for the digits not shown. Since that possibility is extremely unlikely, the calculator check does support our belief that we have the correct solution.

Identities, Conditional Equations, and Inconsistent Equations

An equation that is satisfied by every real number for which both sides are defined is an **identity.** Some examples of identities are

$$3x - 1 = 3x - 1, \qquad 2x + 5x = 7x, \qquad \text{and} \qquad \frac{x}{x} = 1.$$

The solution set to the first two identities is the set of all real numbers, R. Since $0/0$ is undefined, the solution set to $x/x = 1$ is the set of nonzero real numbers, $\{x \mid x \neq 0\}$.

A **conditional equation** is an equation that is satisfied by at least one real number but is not an identity. The equation $3x - 4 = 8$ is true only on condition that $x = 4$, and it is a conditional equation. The equations of Example 1 are conditional equations.

An **inconsistent equation** is an equation that has no solution. Some inconsistent equations are

$$0 \cdot x + 1 = 2, \qquad x + 3 = x + 5, \qquad \text{and} \qquad 9x - 9x = 8.$$

Note that each of these inconsistent equations is equivalent to a false statement: $1 = 2$, $3 = 5$, and $0 = 8$, respectively.

EXAMPLE 2 Classifying an equation

Determine whether the equation $3(x - 1) - 2x(4 - x) = (2x + 1)(x - 3)$ is an identity, an inconsistent equation, or a conditional equation.

Solution

$$3(x - 1) - 2x(4 - x) = (2x + 1)(x - 3)$$

$$3x - 3 - 8x + 2x^2 = 2x^2 - 5x - 3 \qquad \text{Simplify each side.}$$

$$2x^2 - 5x - 3 = 2x^2 - 5x - 3$$

Since the last equation is equivalent to the original and the last equation is an identity, the original equation is an identity.

▸**TRY THIS.** Determine whether $x(x - 1) - 6 = (x - 3)(x + 2)$ is an identity, an inconsistent equation, or a conditional equation. ■

Equations Involving Rational Expressions

Recall that division by zero is undefined and we can't have zero in the denominator of a fraction. Since the rational expressions in the next example have variables in their denominators, these variables can't be replaced by any numbers that would cause zero to appear in a denominator. Our first step in solving these equations is to multiply by the LCD and eliminate the denominators. But we must check our solutions in the original equations and discard any that cause undefined expressions.

EXAMPLE 3 Equations involving rational expressions

Solve each equation and identify each as an identity, an inconsistent equation, or a conditional equation.

a. $\dfrac{y}{y-3} + 3 = \dfrac{3}{y-3}$ **b.** $\dfrac{1}{x-1} - \dfrac{1}{x+1} = \dfrac{2}{x^2-1}$ **c.** $\dfrac{1}{2} + \dfrac{1}{x-1} = 1$

Solution

a. Since $y - 3$ is the denominator in each rational expression, $y - 3$ is the LCD. Note that using 3 in place of y in the equation would cause 0 to appear in the denominators. So we know up front that 3 *is not a solution to this equation.*

$$(y-3)\left(\frac{y}{y-3} + 3\right) = (y-3)\frac{3}{y-3} \quad \text{Multiply each side by the LCD.}$$

$$(y-3)\frac{y}{y-3} + (y-3)3 = 3 \quad \text{Distributive property}$$

$$y + 3y - 9 = 3$$

$$4y - 9 = 3$$

$$4y = 12 \quad \text{Add 9 to each side.}$$

$$y = 3 \quad \text{Divide each side by 4.}$$

If we replace y by 3 in the original equation, then we get two undefined expressions. So 3 is not a solution to the original equation. The original equation has no solution. The equation is inconsistent.

b. Since $x^2 - 1 = (x-1)(x+1)$, the LCD is $(x-1)(x+1)$. Note that using 1 or -1 for x in the equation would cause 0 to appear in a denominator.

$$\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$$

$$(x-1)(x+1)\left(\frac{1}{x-1} - \frac{1}{x+1}\right) = (x-1)(x+1)\frac{2}{x^2-1} \quad \text{Multiply by the LCD.}$$

$$(x-1)(x+1)\frac{1}{x-1} - (x-1)(x+1)\frac{1}{x+1} = 2 \quad \text{Distributive property}$$

$$x + 1 - (x-1) = 2$$

$$2 = 2$$

Since the last equation is an identity, the original equation is also an identity. The solution set is $\{x \mid x \neq 1 \text{ and } x \neq -1\}$, because 1 and -1 cannot be used for x in the original equation.

c. Note that we cannot use 1 for x in the original equation. To solve the equation multiply each side by the LCD:

$$\frac{1}{2} + \frac{1}{x-1} = 1$$

$$2(x-1)\left(\frac{1}{2} + \frac{1}{x-1}\right) = 2(x-1)1 \quad \text{Multiply by the LCD.}$$

$$x - 1 + 2 = 2x - 2$$

$$x + 1 = 2x - 2$$

$$3 = x$$